Linear RF Power Amplifier Design for CDMA Signals

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Abstract

In this paper, we analyze the non-linear effect of an RF power amplifier on CDMA signals and then derive the design formulas which provide the direct relation between the power transistor's traditional non-linearity parameter, the 3rd order intercept point (IP3) and the out-of-band emission levels for CDMA signals. This result will be very useful to system and component engineers in design of RF power amplifiers for CDMA wireless systems.

1. Introduction

Radio frequency resource utilization in mobile cellular communication and satellite communication demands more efficient modulation and multiple access schemes. Qualcomm Inc. has proposed a Code Division Multiple Access (CDMA) scheme with spread spectrum signal and the Federal Communication Committee (FCC) has adopted it as a new wireless communication industry standard, IS-95 [1], in 1993. As in other communication systems, one of the critical components in CDMA system implementation is the RF power amplifiers. A main concern in RF power amplifier design is the nonlinear effect of the amplifier. In the IS-95 standard, FCC has posed a regulation to control the non-linearity of RF amplifiers used in cellular CDMA systems. The non-linearity control regulation is specified by the out-of-band power emission levels [1]; the detail of this regulation will be mentioned in Section 5 of this paper. Traditionally, the non-linearity of an RF amplifier is described by the 3rd order intermodulation coefficient (IP3), or equivalently by the 1 dB compression point [2,3]. Unfortunately, there is no direct link between IP3 and the out-of-band emission levels. This makes it difficult for system designers and RF designers to choose components. To overcome this design obstacle, in this paper, based on the nature of CDMA signals we derive the explicit expressions between the out-of-band power emission levels of an CDMA RF amplifier its given IP3 and output power.

2. Model Description

2.1 Mathematical Model and Its Equivalent of CDMA Signals

A general signal model of a CDMA system with n spread spectrum (SS) signals can be described as

$$s(t)=\sum_{i=1}^{n} m_i(t)c_i(t)\cos(2\pi f_0 t+\theta_i(t)) \qquad (2.1)$$

where $m_i(t)$ is the ith base band signal, $c_i(t)$ is the ith pseudo-noise code with a band width of B, f_o is the center carrier frequency, and $\theta_i(t)$ is the phase of the carrier associated with the ith SS signal.. The signal $s(t)$ has a band width of B which is the bandwidth of the pseudo-noise code [5]. According to the law of large numbers and the central limit theorem in stochastic process analysis [4], the spectrum of $s(t)$ is equivalent to a band-limited white Gaussian stochastic process, as long as the number of SS signals in the CDMA system is large. Hence, statistically we express $s(t)$ equivalently as

$$s(t)=\sqrt{2}x(t)\cos(2\pi f_0 t+\theta) \qquad (2.2)$$

where $x(t)$ is a base band stationary white Gaussian process, and the θ is an arbitrary initial phase. The Power Density Function (PDF) $p_x(f)$ of $x(t)$ is

$$p_x(f)=\begin{cases}\frac{N_0}{2} & |f|\le B\\ 0 & |f|>B\end{cases} \qquad (2.3)$$

where N_0 is a constant. The $p_s(f)$ of $s(t)$ is then [5]

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$$p_s(f) = \begin{cases} \frac{N_0}{4} & |f - f_0| \le B \\ 0 & |f - f_0| > B \end{cases} \quad (2.4)$$

The power of *s(t)* can be obtained from $p_s(f)$:

$$P_s = \int_{-\infty}^{\infty} p_s(f)\, df = N_0 B \quad (2.5)$$

Since we are only interested in the pass-band behavior of *s(t)*, the spectral analysis to be conducted in this paper can be done based on the low pass equivalent, or the complex envelope, of *s(t)* [5]. Denote $\tilde{s}(t)$ as the low pass equivalent of *s(t)*. From eq.(2.2) we can write

$$\tilde{s}(t) = x(t). \quad (2.6)$$

2.2 Mathematical Model of a Power Amplifier

A general Taylor series model for an power amplifier has been well accepted in linear amplifier design[2,3], although there exist some more comprehensive models [6]. Using the low pass equivalent $\tilde{s}(t)$, the output low pass equivalent of the amplifier can be expressed as

$$y(t) = F(\tilde{s}(t)) = \sum_{i=0}^{\infty} a_{2i+1} \tilde{s}^{(2i+1)}(t) \quad (2.7)$$

where only the odd terms in the Taylor series are considered since that the spectrums generated by the even terms are at least f_0 away from the center of the pass band and for $B << f_0$, the effects from these terms to the pass band are negligible. Furthermore, as a linear amplifier the third order term is the dominate term in (2.7), assuming that the fifth order is insignificant. Therefore, in the analysis to be presented, we will use the following modeling for an RF amplifier:

$$y(t) = F(\tilde{s}(t)) = a_1 \tilde{s}(t) + a_3 \tilde{s}^3(t). \quad (2.8)$$

In this equation, a_1 is the linear gain G of the amplifier and a_3 is a non-linear coefficient directly related to the third order intercept point IP3. For a linear power amplifier, a_1 and a_3 can be expressed as [2]

$$a_1 = 10^{\frac{G}{20}} \quad (2.9)$$

and

$$a_3 = \frac{2}{3} 10^{(\frac{-IP3}{10} + 3\frac{G}{20})} \quad (2.10)$$

Through eqs. (2.8), (2.9), and (2.10), we see that an amplifier's output $y(t)$ is a function of G and IP3 and the input signal $\tilde{s}(t)$. Consequently, using eq. (2.8) and the stochastic property of $\tilde{s}(t)$, we can calculate the spectrum of $y(t)$ as well as the power emission levels. Therefore we are able to evaluate all the non-linear effects of the amplifier to CDMA signals.

3. Calculation of the Spectrum of Amplifier's Output $y(t)$

We employ the Wiener -Khintchine theorem [5] to calculate the spectrum of $y(t)$, denoted by $P_y(f)$. According to the theorem, the spectrum $P_y(f)$ is related to the correlation function $R_y(\tau)$ of $y(t)$ by

$$P_y(f) = \int_{-\infty}^{\infty} R_y(\tau) e^{-j2\pi f \tau} d\tau. \quad (3.1)$$

By definition, $R_y(\tau)$ is expressed as

$$\begin{aligned} R_y(\tau) &= E\{y(t)y(t+\tau)\} \\ &= E\{[a_1\tilde{s}(t) + a_3\tilde{s}^3(t)][a_1\tilde{s}(t+\tau) + a_3\tilde{s}^3(t+\tau)]\} \\ &= a_1^2 E\{\tilde{s}(t)\tilde{s}(t+\tau)\} + a_1 a_3 E\{\tilde{s}(t)\tilde{s}^3(t+\tau)\} \\ &\quad + a_1 a_3 E\{\tilde{s}(t+\tau)\tilde{s}^3(t)\} + a_3^2 E\{\tilde{s}^3(t)\tilde{s}^3(t+\tau)\} \end{aligned} \quad (3.2)$$

where $E\{\cdot\}$ is the expectation of $\{\cdot\}$. We now calculate the correlations in eq. (3.2). Since from eq.(2.6) $\tilde{s}(t)$ is a white Gaussian stationary process, the first term of eq. (3.2) is just the correlation of $\tilde{s}(t)$ which can be written as [5]

$$E\{\tilde{s}(t)\tilde{s}(t+\tau)\} = \frac{N_0 \sin 2B\tau}{2\pi\tau}. \quad (3.3)$$

The expectations of $\tilde{s}(t)\ \tilde{s}^3(t+\tau)$ and $\tilde{s}^3(t)\ \tilde{s}(t+\tau)$, which are the second and third terms of eq. (3.2), can be calculated by the formula [5]

$$E\{X_1 X_2 X_3 X_4\} = \sigma_{12}\sigma_{34} + \sigma_{23}\sigma_{14} + \sigma_{24}\sigma_{13} \quad (3.4)$$

where X_i are zero mean Gaussian variables and σ_{ij} are the correlations of $X_i X_j$. Using eqs. (3.3) and (3.4), we obtain

$$E\{\tilde{s}(t)\tilde{s}^3(t+\tau)\} = E\{\tilde{s}^3(t)\tilde{s}(t+\tau)\} = 3N_0B\frac{N_0 \sin 2\pi B\tau}{2\pi\tau} \quad (3.5)$$

The calculation of the last term in eq. (3.2) is quite involved. By using the probability density function of a multiple Gaussion [4] and applying a tedious manipulation, we get

$$E\{\tilde{s}^3(t)\tilde{s}^3(t+\tau)\} = \frac{9}{2}N_0^2B^2\frac{N_0 \sin 2\pi B\tau}{2\pi\tau} + 3\left(\frac{N_0 \sin 2\pi B\tau}{2\pi\tau}\right)^3 \quad (3.6)$$

Thus, $R_y(\tau)$ is written as

$$R_y(\tau) = (a_1^2 + 6a_1a_3BN_0 + \frac{9}{2}a_3^2N_0^2B^2) \times \frac{N_0 \sin 2\pi B\tau}{2\pi\tau} + 3a_3^2\left(\frac{N_0 \sin 2\pi B\tau}{2\pi\tau}\right)^3 \quad (3.7)$$

Using eq. (3.1) and the corresponding relation of time domain product and frequency domain convolution, we have

$$P_y(f) = \int_{-\infty}^{\infty} R_y(\tau)e^{j2\pi\tau}d\tau = \begin{cases} (a_1^2 + 6a_1a_3BN_0 + \frac{9}{2}a_3^2N_0^2B^2)\frac{N_0}{2} \\ +3a_3^2 P_{\tilde{s}}(f)\otimes P_{\tilde{s}}(f)\otimes P_{\tilde{s}}(f) & |f| \le B \\ 3a_3^2 P_{\tilde{s}}(f)\otimes P_{\tilde{s}}(f)\otimes P_{\tilde{s}}(f) & |f| > B \end{cases} \quad (3.8)$$

where $P_{\tilde{s}}(f)$ is the PDF of $\tilde{s}(t)$ and "$\otimes$" denotes convolution. Since $P_{\tilde{s}}(f)$ is a rectangular function, the calculation of its convolution is straightforward, we finally obtain

$$P_y(f) = \int_{-\infty}^{\infty} R_y(\tau)e^{j2\pi\tau}d\tau = \begin{cases} (a_1^2 + 6a_1a_3BN_0 + \frac{9}{2}a_3^2N_0^2B^2)\frac{N_0}{2} \\ +3a_3^2\frac{N_0^3}{4}(2B^2 - f^2) & |f| \le B \\ 3a_3^2\frac{N_0^3}{4}(2B - |f|)^2 & B < |f| \le 3B \end{cases} \quad (3.9)$$

4. The Out-of-band Spurious Emission Power in a Certain Frequency Band

Let the frequency band be defined by f_1 and f_2, where both f_1 and f_2 are larger than B. With the explicit form of $P_y(f)$, the out-of-band spurious emission power level within the band (f_1, f_2) is determined by

$$P_{IM3}(f_1,f_2) = \int_{f_1}^{f_2} 3a_3^2\frac{N_0^3}{16}(3B - |f|)^2 df = a_3^2\frac{N_0^3}{16}[(3B - f_1)^3 - (3B - f_2)^3] \quad (4.1)$$

However this expression is not easy to use, because it is not tied to IP3 and the linear output power P_0 of the amplifier. By using eq.(2.10) and noticing that the product $a_1^2N_0B$ is just the linear power P_0 of the amplifier, we can replace a_3 by IP3 (dBm) and P_0 (W), yielding

$$P_{IM3}(f_1,f_2) = 0.028\cdot 10^{\frac{-(IP3-30)}{5}} P_0^3 \times \frac{[(3B - f_1)^3 - (3B - f_2)^3]}{B^3} \text{(W)} \quad (4.2)$$

In most design procedures, one is concerned with the required IP3 for a given output power under a certain level of $P_{IM3}(f_1,f_2)$. To obtain the required IP3, we can just solve eq.(4.2) for IP3 for a given $P_{IM3}(f_1,f_2)$:

$$IP3 = -5\log\left[\frac{P_{IM3}(f_1,f_2)B^3}{P_0^3[(3B - f_1)^3 - (3B - f_2)^3]}\right] + 22.2 \quad \text{(dBm)} \quad (4.3)$$

This result gives the direct relation between the out-of-band emission level of a CDMA signal power amplifier and its IP3. With a given required IP3, the power amplifier design for a CDMA signal becomes a conventional RF power amplifier design.

5. Example and Comparisons with Real Measurements

5.1 Design Example

In this example, we use the result shown in eq. (4.3) to design an amplifier of 18 W which complies with the out-of-band emission level control requirement of IS-95. The two out-of-band emission level control requirements are recalled in the following:

(1) The total CDMA signal bandwidth is 1.226 MHz; then $B = 613$ KHz. In the bands of [f_0, f_0+ 750 KHz] and [f_0 - 750 KHz , f_0], the compression level between the output power and the out-of-band emission

power at of 30 KHz bandwidth must be larger than 45 dBc.

(2) For offset frequencies greater than 1.98 Mhz from f_0, the spurious emission power of a 30 KHz band must be less than 60 dB.

For this amplifier, $P_0 = 18$ W. For Condition 1, the corresponding maximum $P_{IM3}(f_1, f_2)$ is 5.7×10^{-4} W. Then, from eq. (4.3) the required IP3 comes as

$$IP3 = -5\log\left[\frac{0.00057 \times 625^3}{18^3[(1839-735)^3 - (1839-765)^3]}\right] \quad (5.1)$$
$$+22.3 = 56 \text{ dBm}$$

For Condition 2, due to the fact that the only third order intermodulation is counted in the analysis, there is no out-of-band emission power at a frequency 3 B away from the center frequency f_0. Therefore, the above results indicate that in order to meet the IS-95 requirement, the 18W CDMA amplifier must have an IP3 of at least 56 dBm.

5.2 Comparison with Real Measurements

Several measurements of the out-of-band emission levels of CDMA signals have been done by Qualcomm using a PA9440 amplifier designed by Clewave. The measured compression levels at the frequency f_0+750 KHz are shown in Figure 1. Also in Figure 1, the calculated levels from Equation (4.2) are shown by the solid line. As seen in the figure, the real measurements and the analytical results are very close for $P_0 > 25$ dBm. The bigger difference occurs for $P_0 < 25$ dBm, which is due to the fact that $P_{IM3}(f_1, f_2)$ is not significant and is comparable to the noise floor.

6. Conclusions

In this paper, by proposing a theoretical method to predict the output power spectrum of a CDMA power amplifier, we directly link the traditional linearity parameter IP3 with the out-of-band emission levels. This makes it possible for RF power amplifier designers to use the conventional approach to design the RF power amplifier for CDMA signals. However, it should be pointed out that in the derivation, we have ignored the effect of the fifth order intermodulation. This could affect the accuracy of the formulas when the fifth order intermodulation is high, such as when GaAs MESFETs are used. Although it is possible to obtain an analytical result, including the higher order intermodulation terms, the result may not be convenient to use, since the fifth intermodulation level is not given in transistor's data books. Generally speaking, with the fifth intermodulation down 20 dB from the third, the design formula (4.3) can provide a good approximation with an error within 1 - 1.5 dBm. Being aware of the limitations, one may be able to use the obtained formulas more efficiently for linear CDMA power amplifier design.

Acknowledgment

The authors wish to thank their colleagues: David Wills and Scott Hazenboom for the insightful discussions and comments.

7. Reference

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Fig. 1 Comparison of Measurements and Calculation